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# AN IMPROVED MULTIMODE SMALL APERTURE/OBSTACLE THEORY

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## ABSTRACT

We present a new and improved theory for small apertures and obstacles in multimode waveguide, and we show that for transverse discontinuities the resulting equivalent network differs from that obtained from earlier theory by simple added series or shunt elements. That simple network modification, however, yields greatly improved accuracy, as demonstrated in a specific example.

## 1. INTRODUCTION

Small aperture/obstacle theory is one of the gems of microwave field theory in that it allows one to derive, with very little work, very simple analytical expressions for certain classes of waveguide discontinuities. In fact, it permits one to establish simple recipes for such derivations. In addition, those expressions have been found to yield accurate numerical values even somewhat outside their expected range of validity.

The last of the above remarks (accurate values) seems so far to be true only for discontinuities in a waveguide that can support only a single mode. Small aperture/obstacle theory has been extended to the multimode waveguide case [1], and it was found that simple expressions could still be obtained. When we tried to use those expressions, however, we found that they were much less accurate than corresponding single-mode expressions are known to be.

To understand the added difficulty introduced when a multimode situation is addressed, let us first consider a small hole in a transverse metal wall in a waveguide. When a waveguide mode is incident on that small hole, the hole is replaced, in small aperture theory, by a magnetic dipole and/or an electric dipole located on the wall. These dipoles are then characterized in terms of polarizabilities related to the size and shape of the hole. The hole is regarded as small in two basic ways: small compared to the waveguide cross-section dimensions, and small compared to the wavelength; these polarizabilities can thus be taken corresponding to apertures in free space and at zero frequency, respectively. The simplicity in small aperture theory occurs because the expressions for the polarizabilities in free space and under static conditions become particularly simple.

We now return to multimode conditions. The size of the hole relative to the cross-section dimensions is a function of geometry and is not affected by whether the waveguide can carry one or more than one propagating mode. On the other hand, the hole size relative to free-space wavelength $\lambda_o$ is indeed affected by such considerations. When a waveguide can support several propagating modes, its cross section must be substantially larger than $\lambda_o$ even though this is not so for a single-mode waveguide. Thus, a hole that was small compared to $\lambda_o$ when the guide supported only one propagating mode may no longer be small in that sense when the guide supports several propagating modes. In essence, the range of validity of the solution becomes substantially reduced.

In our new, improved solution, this limitation is lifted and the results are valid over a much larger range of aperture/obstacle dimensions. Our improved theory shows that for transverse discontinuities the resulting equivalent network differs from that obtained from earlier theory by simple added series or shunt elements related directly to the modal static characteristic impedances. That simple network modification, however, yields greatly improved accuracy. The fact that this improvement in accuracy can be obtained by means of a simple network modification makes our improved theory particularly attractive.

In Sec. 3 below we compare numerical values derived by the earlier theory with those obtained by our improved theory for a specific example, namely, plane-wave scattering by a multimode metal-strip grating. The improvement introduced by the simple network modification is seen to be dramatic.

## 2. OUR IMPROVED FORMULATION

In the customary small aperture/obstacle formulation, and also in its extension to multimode situations [1], the dynamic kernel in the relevant integral equation is effectively replaced by a static one. This replacement is an approximation intended to correspond to the usual static phrasings in small aperture/obstacle theory. Our new formulation also produces static kernels for the relevant integral equations, but we keep the complete integral equations



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rigorous by appropriately redefining the modal voltages and currents [2]. Then, when we solve the relevant integral equation in the small argument limit, the approximation involves only the ratio of the aperture/obstacle size to the cross-section dimensions, and does not relate to $\lambda_o$. Our new formulation does not require the size to be small relative to $\lambda_o$.

As a direct result of the redefined modal voltages and currents, the equivalent networks that follow from our formulation contain small additional shunt or series elements (depending on whether we are treating an aperture or an obstacle, respectively) that are proportional to the modal static characteristic impedances.

For the case of a transverse obstacle in a waveguide, the equivalent network is in shunt across the transmission lines representing the propagating modes. Reference 1, in its Fig. 3, presents the general form for this network; that network is basically reproduced in Fig. 1 here, but we retain only three modes for clarity. Our new formulation yields the modified network shown in Fig. 2, where again only three modes are retained. It is seen that the two networks differ only in the series elements $(-Z_{ns}/2)$ for each of the higher modes n that appear in Fig. 2 but are missing from Fig. 1. The terms $Z_{ns}$ are the static modal characteristic impedances

$$Z_{ns} = \begin{cases} \dfrac{\lim\limits_{\omega\to 0}\beta_n}{\omega\epsilon_o\epsilon_r} & \text{for TM modes} \\[2ex] \dfrac{\omega\mu_o\mu_r}{\lim\limits_{\omega\to 0}\beta_n} & \text{for TE modes} \end{cases}$$

and $\beta_n$ is the propagation wavenumber of the $n^{th}$ mode. The modification in the network thus appears to be minor, and is simple in form, but it greatly improves the accuracy of the result, as shown in the next section.

If the corresponding analysis is performed for a transverse aperture, the added elements in each higher mode line are in shunt rather than in series, and each element is $(-2Y_{ns})$, where $Y_{ns}$ is the static modal characteristic admittance.

## 3. NUMERICAL COMPARISONS

In order to demonstrate the importance of the seemingly small modifications to the network, and to show that the new network is dramatically more accurate, we next present numerical comparisons for a specific structure. The structure is the grating of small strips shown in Fig. 3, upon which a plane wave with TM polarization is incident at an angle of 15°. We present numerical results for the reflected powers in the n= 0 (incident) mode and in the n= 1 (third) mode, for two different ratios of d/p, the strip width d of the obstacle to the period p of the grating. The computed values using the networks of Figs. 1 (old theory) and 2 (new theory) are compared with values obtained from an independent reference solution, known to be accurate [3].

For the structure shown in Fig. 3, the networks in Figs. 1 and 2 become particularly simple in form in the small obstacle limit. Figure 2, which incorporates the modifications required by our improved theory, then takes the form shown in Fig. 4. Again, however, the networks corresponding to the old and new theories differ only by the added series elements that appear in Fig. 4.

We should indicate next that the expression for the element $Y_{11}$ in Fig. 4 is found from both the old and the new theories to be [3]

$$Y_{mn} = Y_{11} = j\omega\epsilon_o\epsilon_r\frac{\pi}{4}p\left(\frac{d}{p}\right)^2$$

The added elements in Fig. 4 are

$$-\frac{1}{2}Z_{ns} = j\frac{\pi\,|n|}{\omega\epsilon_o\epsilon_r p}$$

It is evident that the elements that comprise the network in Fig. 4 are particularly simple.

In the computations mentioned above, we considered for this comparison only three modes, the n= 0, ± 1, consistent with the network in Fig. 4. The reflected powers in the incident (n= 0) mode are plotted in Figs. 5 and 6 as a function of $p/\lambda_o$; the unlabeled curve corresponds to the accurate reference solution, and the other two are labeled. The ratios of d/p are 0.1 and 0.3. As $p/\lambda_o$ increases, we note that first only the incident mode can propagate, then the n= -1 mode is also above cutoff, and finally all three modes (n= 0, n= -1 and n= 1) propagate. (Higher propagating modes correspond in the diffraction context to propagating spectral orders.) The regions are separated by cusps in the curves.

For the d/p = 0.1 case, the obstacle is really very small, and results from both the old theory and the new theory agree well with the reference solution, although the new theory is clearly in better agreement. For d/p = 0.3, a much larger obstacle, it is seen that the new theory is dramatically better than the old one, which is evidently quite far out of its range of validity. The agreement between the new theory and the reference solution is excellent when two modes are propagating, but is less good when all three modes are above cutoff. The agreement should improve when four modes are included in the network.

The same qualitative agreement is found in Figs. 7 and 8, where the reflected power in the n= 1 higher mode is plotted vs. $p/\lambda_o$. For d/p = 0.1, the agreement is quite good for both the old and new theories, but for the larger obstacle size, d/p = 0.3, the new theory is clearly far superior.

In summary, we have developed a modified formulation for multimode small aperture/obstacle theory, and we have shown that for transverse obstacles the resulting equivalent network differs from that obtained from the earlier theory by simple added series elements. That simple network modification, however, yields greatly improved accuracy, as demonstrated in a specific example.

## References


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[2] N. Marcuvitz, Waveguide Handbook, Vol. 10, MIT Radiation Laboratory Series, McGraw-Hill Book Co., New York, 1951, p. 147.

[3] M. Guglielmi, "Radiation from a Metal Strip Grating on a Dielectric Slab," Ph.D. Dissertation, Polytechnic University, June 1986.


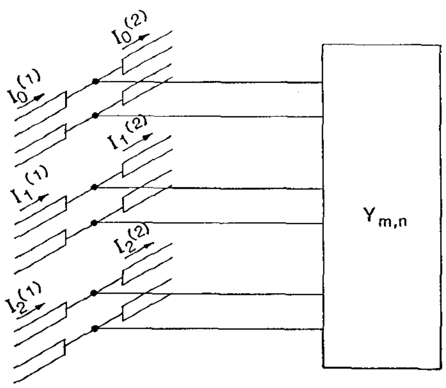


Fig. 1 General form of the equivalent network for a transverse multimode obstacle, given in Ref. [1].

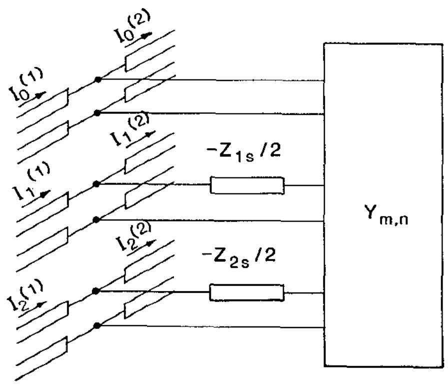


Fig. 2 Equivalent network corresponding to Fig. 1, but obtained by using our improved theory. Note that it differs from the network in Fig. 1 by the presence of added series elements associated with the higher modes.

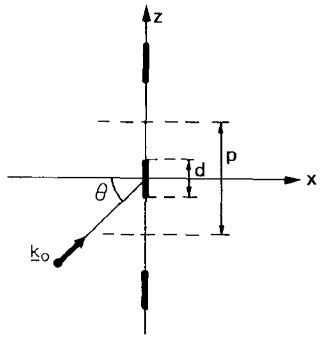


Fig. 3 Periodic grating of small metal strips, upon which a plane wave is incident at an angle.

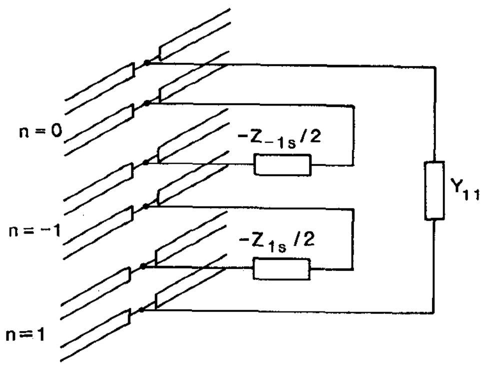


Fig. 4 Equivalent network to which the network in Fig. 2 reduces when it is applied to the grating in Fig. 3. The network is seen to become particularly simple in form.

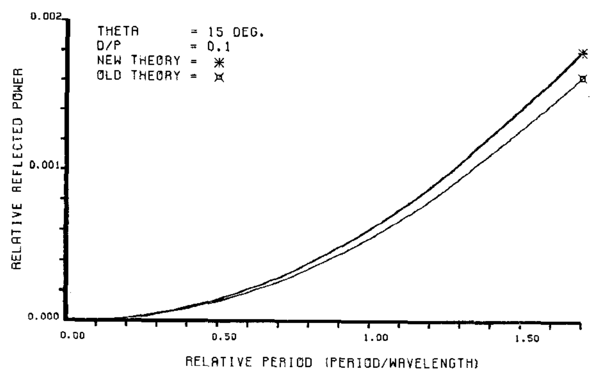


Fig. 5 Reflected powers in the incident (n= 0) mode as a function of relative period $p/\lambda_o$ for relative obstacle width $d/p$ = 0.1. The unlabeled curve refers to the accurate reference solution, and the other two curves correspond to the old and new theories, as labeled.

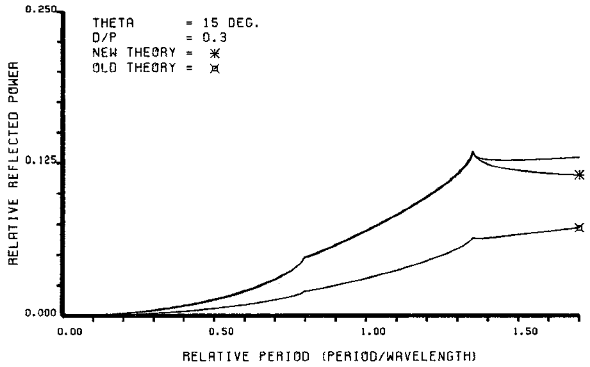


Fig. 6 Same as Fig. 5 but for d/p = 0.3, a much larger obstacle.

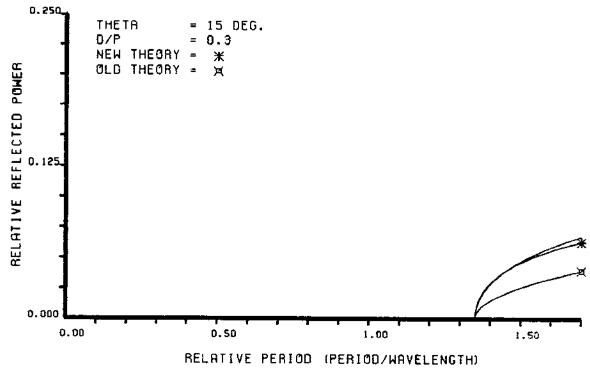


Fig. 8 Same as Fig. 7 but for d/p = 0.3, a much larger obstacle.

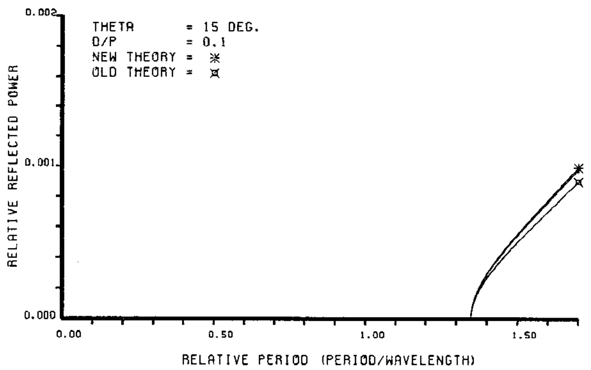


Fig. 7 Reflected powers in the n=1 higher mode as a function of relative period $p/\lambda_0$ for relative obstacle width d/p = 0.1. The unlabeled curve refers to the accurate reference solution, and the other two curves correspond to the old and new theories, as labeled.

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